THE ANALYSIS OF TUNED MASS DAMPERS OF TAIPEI 101 TOWER

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ABSTRACT

The tuned mass dampers (TMDs) are the passive energy dissipation devices. Their application as dampers specially in the tall buildings and sky scrapers etc. has become significantly important. One of the applications is the Taipei-101 tower. This paper discusses the response of the building and the absorber under different forcing functions. Two models of the system are made. First a system as two masses and two degrees of freedom and second a system with three masses and three degrees of freedom. The system is modeled using the state space approach, which makes it easier and quicker to compute the responses. The response under the approximate earthquake model is also discussed. The results highlight the importance of the TMDs in the building in sustaining the impact loads, wind gales and random forcing functions. The effect of changing the stiffness and damping coefficients of the TMD are also discussed. The results can be used to extend the work to a higher degree of freedom systems.

Keywords:
TMD, Energy Dissipation, State Space
INTRODUCTION
The modern structural protective systems are divided into three general groups;

1. **Seismic isolations**
   - Elastomeric bearings
   - Lead rubber
   - Combined elastomeric and sliding bearings
   - Sliding friction pendulum systems
   - Sliding bearings with restoring force

2. **Passive energy dissipation**
   - Metallic dampers
   - Friction dampers
   - Viscoelastic solid dampers
   - Viscoelastic or viscous fluid dampers
   - Tuned mass dampers
   - Tuned liquid dampers

3. **Semi-active and active systems**
   - Active bracing systems
   - Active mass dampers
   - Smart materials
   - Variable stiffness and damping systems

Our work will be focused on the tuned mass dampers. The reason is the effectiveness of TMDs in reducing the wind and earthquake induced structural vibrations especially in the tall buildings like skyscrapers and chimneys and bridges etc. One of the examples is the damper system in the Taipei 101, which can withstand wind gales up to 60 meters/sec.

It is one of the largest TMDs with a weight of 660 tons and 5.5m diameter shown in the figure. It is suspended from the 92nd floor to 87th floor.

![Figure 1: The TMD used in Taipie-101.](image1.png)

It consists of a pendulum system. The figure shows the massive ball used in the system. It has springs on all its sides which are attached with the floor walls from the other side. These springs are used to minimize the amplitude of vibration of the main system/ball.

It is generally implied at the places where the structural deflection is maximum.

The natural frequency of the damper is tuned so that under an excitation the damper is out of phase with the structure and goes to resonance itself, thus dissipating the energy.

The TMD concept was first applied by Frahm in 1909 to reduce the wobble in ships as well as ship hull vibrations. The figure below shows a general Frahm’s absorber. It is an undamped system, representing the general principal on which a TMD works.

![Figure 2: Frahm’s Absorber (general).](image2.png)

It contains a small mass ‘m’ attached to a spring with stiffness ‘k’ and a large mass ‘M’ attached to a spring with stiffness ‘K’. It can be proved that under a general harmonic load on the main mass ‘M’, if the natural frequency of the small mass ‘m’ i.e. is equal to the excitation frequency, \( \sqrt{(k/m)} \), is equal to the excitation frequency, then the large mass ‘M’ can be kept stationary while the small mass ‘m’ will resonate.
LITERATURE REVIEW

Henri P. Gavin [1] has discussed the behavior of the TMDs. He has derived an extensive analytical algorithm to study the different types of TMD systems. His work is significant because of the derived algorithm which can help in solving the TMDs for the tall buildings.

Prof. Yogesh Ravindra Suryawanshi, Prof. Amar shitole and Prof. D.T. Rahane [2] studied of TMD Systems for the vibrational control of multistoried buildings. Their work is significant because they have studied the effect of TMD in tall buildings and analyzed the effectiveness of TMDs to preserve the structural integrity.

Rashmi Mishra [3] studied about the vibration control of frame structures under seismic excitations with applications of TMDs. She has investigated the use of TMDs in vibration control of tall and light buildings which are also very flexible and have a very low value damping value. Her work is significant because she has derived a numerical algorithm of a tall building installed with TMD and then analyzed it.

L. Zuo and S. A. Nayfeh [4] have discussed the design of MDOF absorber for the damping of MDOF systems. They have described the optimization of multi degree of freedom TMDs and derived expressions regarding optimization of minimum damping modes in given frequency range for a general MDOF system.

Gebrail Bekdas and Sinan Melih Nigdeli [5] described the use of optimum tuned mass dampers for structures with different damping and periods. They have described the need of optimization of TMDs for use in megastructures as it would lower the cost and give the best performance. Their work is significant because they have derived how to optimize the TMDs and simplified it for future application.

C. Lindh, S. Laflamme and J. J. Connor [6] studied the performance of semi active TMDs because of the nonlinear damping device effects. Their work is significant because they have described the effectiveness of use of different types of damping devices such as variable orifice (VO) and magnetorheological (MR) for the semi active control of tuned mass dampers. They have also investigated the effect of nonlinear damping devices on the control of semi active TMDs.

Seyed Mehdi Zahrai and Amin Ghannadi Asl [7] discussed about how tuned mass dampers causes reduction in the seismic response of tall buildings. They have discussed the effectiveness of TMDs in controlling the vibrations of tall buildings. They have discussed the practical considerations and vibration control effectiveness of the TMDs of 15, 20, 25 and 30 story buildings. Their work is significant because they have simplified the use of TMDs and their analysis for up-to 30 story buildings.

Dr. Mohan M. Murudi and Mr. Sharadchandra M. Mane [8] studied the tuned mass damper (TMD) with seismic effectiveness for different ground motion parameters. Their research is significant because of investigation they have conducted about the effectiveness of TMDs in controlling the seismic response of structures, wind induced vibrations and ground motions. They have considered a SDOF system for their experiments and conducted different tests on this system. They have found that TMDs are very effective in controlling earthquake response of lightly damped structures. However, effectiveness of TMDs is not affected by the intensity of earthquake.

Lei Zuo and Samir A. Nayfeh [9] described the suppression of single-mode vibration of the multi degree of freedom tuned mass damper which are under random and harmonic excitation. Their work is significant because of the fact that they utilized different modes of vibration of a body relative to a primary structure be tuned to one natural frequency of the primary system. They have proved the effectiveness of 2DOF absorber over SDOF absorber.
METHODOLOGY

All vibrating structures dissipate energy due to the presence of the internal stresses, deformations etc. the higher the ability of a structure to dissipate energy the lower will be the resulting amplitude of vibrations. This property of dissipating the energy can be enhanced by the use of different dampers which does not depend on the external forces. One of these types of dampers is called the tuned mass damper systems or TMDs. These systems are well utilized in reducing the wind induced vibrations, especially in the tall buildings like skyscrapers, bridges etc.

The building under discussion could be modeled as a minimum of two degrees of freedom (DOF) considering only the lumped mass of the building and mass of the absorber. We could also model the system as separate masses for each of the story. Such a system will have 101 masses for the stories and one mass of the absorber, a total of 102 masses so 102 DOFs.

We will study our system in two different ways.

Two Masses:

First of all, the system is modeled as two masses; one main mass of the building and second the mass of the absorber. The system will look like as shown in the figure;

![Figure:3. Two masses model.](image)

The equations of motion for this system are;

\[
\begin{align*}
    m_1 \ddot{x}_1 + k_1 x_1 + k_2 (x_1 - x_2) + c_2 (\dot{x}_1 - \dot{x}_2) &= f_1 \\
    m_2 \ddot{x}_2 + k_2 (x_2 - x_1) + c_2 (x_2 - x_1) &= f_2
\end{align*}
\]

Where 'm1' is the mass of the building, 'm2' is the mass of the absorber, 'k1' is the stiffness for the building, 'k2' is the stiffness for the absorber and 'c2' is the damping for the absorber. Applied forces are represented by 'f1' and 'f2' for mass '1' and mass '2' respectively. The damping for the main building is considered to be zero so that it is assumed that all the vibrations are damped out using the TMD.

Three Masses:

The system is modeled as three masses or say three degrees of freedom. The main mass of the building as considered for the above problem is then divided in two parts. We know that the damper (TMD) is positioned at the 88th floor. Then the system takes the form as shown in the figure below;

![Figure:4. Three masses model.](image)

In the above figure the mass 'm1' is the mass of the stories from 88th to 101st for our under consideration building “Taipei 101”. The mass ‘m2’ is the mass of the absorber/TMD. The mass ‘m3’ is the mass of the building for the stories from 1st to 88th floor. The equations for this system are;

\[
\begin{align*}
    m_3 \ddot{x}_3 + k_3 x_3 + k_2 (x_3 - x_2) + c_2 (\dot{x}_3 - \dot{x}_2) &= f_3 \\
    m_2 \ddot{x}_2 + k_1 (x_2 - x_1) + k_2 (x_3 - x_2) + c_2 (\dot{x}_1 - \dot{x}_2) &= f_2 \\
    m_1 \ddot{x}_1 + k_1 x_1 + k_2 (x_2 - x_1) &= f_1
\end{align*}
\]
For these equations mases, stiffness’s and damping constants are as shown in the figure. The subscript ‘3’ is for the first 88 floors, ‘2’ is for the absorber and ‘1’ is for the floors from 88th to 101st.

**State Space Representation:**

We model both of our systems using the state space representation. The damping of the building is considered to be zero so that all the energy is dissipated using the TMD. Modeling the system and writing in general space state form as;

\[ \dot{x} = Ax + Bu \]

Where \( x \) is the state vector, \( u \) is the input/force vector, \( A \) is the state/system matrix and \( B \) is the input/force matrix. The general equation of the output is;

\[ y = Cx + Du \]

Where \( y \) is called the output vector, \( C \) is the output matrix and \( D \) is the feed through matrix. It shows the direct relation of output with the input. The matrix \( D \) in our case is zero because the system model does not have a direct feed through or output is not directly affected by the input.

When the system has been defined in the space state form the response of the system is determined for different forcing functions.

**Two Masses**

First of all, the system of two masses is studied under different forcing conditions.

1. **Impulse Response:**

   The response of the system is plotted for a unit impulse applied on the two degree of freedom system.

![Figure:5(a). Unit impulse response of the system, mass-1/building as out(1) and mass-2/absorber as out(2).](image)

As a response to the unit impulse the main mass has a decaying response and in the first 1.5 seconds the motion is completely damped out, while the absorber keeps on oscillating with a very small amplitude. The absorber comes to its rest position at 35.2 seconds after the load has been applied as shown in the following figure.

![Figure:5(b). Absorber's oscillations damping out (y-axis is in 10^-12).](image)

2. **Unit Step Response:**

   A unit step is applied and the response is plotted as shown in the figure below;
3. **Step Response for a Short Period:**

The response of the system is plotted for a step input for 3 seconds.

**Figure:7.** Input Heaviside of magnitude 400 for 3 seconds

The step input of 400 magnitudes is applied to our system for 3 seconds and the system response is as shown in the following figure.

These plots show that as soon as the force is removed from the building the response dies out in the next 1 second and the building comes to rest all owing to the absorber which comes to rest at around 27 seconds.
4. Earthquake Response:

We have attempted to model the earthquake as an input with continuous changing amplitude as shown in the following figure.

![Figure:10. Approximate Earthquake function.](image)

If an input of this is applied to the system only for the first 3 seconds as shown in the figure below.

![Figure:11. Earthquake input for 3 seconds.](image)

The response of the system is plotted w-r-t time as illustrated in the following figure.

![Figure:12. Response of the building to an earthquake input.](image)

5. Input Dying Sinusoid:

An input of dying sinusoid is applied at the system which is terminated at nine seconds. The input is shown in the figure.

![Figure:14. Dying sinusoid.](image)

These figures show that the building vibrates in response to the input for 7 seconds and then comes to rest while the absorber keeps on oscillating and dissipating the energy from the system.

![Figure:13. Response of the absorber to an earthquake input.](image)

![Figure:15. Response of the building to a dying sinusoid.](image)
The absorber comes to rest after almost 200 seconds from the time when the load was applied.

An important phenomenon is to be observed here that the system keeps on vibrating with the same frequency as the input until the input is removed. After the input is removed it takes no time for it to settle down.

Three Masses

Here the system is modeled for three masses; one main mass of the system is the mass of stories from 89 to 101 \((m_1)\), second the mass of the absorber and third is the mass of first 88 stories \((m_3)\). The damping of the building is considered to be zero so that all the energy is dissipated using the TMD.

We will study the response under different forcing conditions.

1. Unit impulse Response:

What if there is a blast just at the 88th floor which is the position of our damper hanging from the top floor or in other case we can say that direct impact on the mass ‘\(m_2\)’ of our system. The response of such a system will be as given for a unit impulse in the following figure.

![Figure 17](image)

**Figure 17.** Response of masses out-1\((m_1)\), y-scale \((10^{-8})\), out-2\((m_2)\), y-scale \((10^{-8})\) & out-3\((m_3)\), y-scale \((10^{-10})\) for a unit impulse applied at ‘\(m_1\)’.

The displacement amplitude shows that the top floors will have the same amplitude as of the absorber which had a direct impact. This is dangerous for the building. While at the bottom floors very little impact will be felt, thus very small amplitude of vibrations.

2. Unit Step Response:

This can be realized as a constant input force at the top floors of the building, which might be because of constant wind gust or a passing by
aircraft which is generally the case for very tall buildings. This response can be studied by modeling the response under a unit step input at the mass ‘m1’.

Figure:18. Response of masses out-1(m1), out-2(m2) & out-3(m3) for a unit step applied at ‘m1’.

The displacement at the bottom stories is much smaller as compared to the top stories which is clear from the response of the mass ‘m3’ i.e. in $10^{-11}$ meters.

3. Dying Sinusoid:

Let’s assume a sinusoidal pulse is transferred from the ground directly to the ground floors or in our case mass ‘m3’. If the pulse acts for 9 seconds and then finishes suddenly, the forcing function will then be;

Figure:19. Dying sinusoid (Forcing Function)

The response of the system to such forcing function was also discussed for two masses system. The response of a three mass system is shown in the following figures.

Figure:20. Response of the stories from 1st to 88th.

Figure:21. Response of the stories from 88th to 101st.
Again the same behavior, as long as the forcing function is acting the system will vibrate and the motion is analogous to that of the input. But as the input is removed all the three masses come almost to the equilibrium position in next 10 seconds.

4. Earthquake Response:

An attempt has been made to model the earthquake response of the system. The force input shown in the following figure is taken as an approximate seismic function.

This approximate earthquake forcing function is applied to the system under consideration. Note one thing that for the under mentioned section this load will be applied at the mass 'm3' only i.e. at the bottom 88 stories. The load will be applied for 3 seconds only and the response of the system is plotted.
Effects of Changing the Absorber Damping:

The responses for all three masses are more or less similar to each other for the time five times the time for which forcing function is applied, though their magnitudes are different. This is the first shock. After the first shock which stays for 15 seconds, the absorber starts its job and the oscillations are damped out in about 30 seconds. If we take the damping of the absorber to be zero, then the oscillations will continue with the same magnitude and amplitude as was set at the end of the first shock. This is shown in the figure below for the bottom stories.

Figure:27. Response of the stories 1st to 88th with zero absorber damping.

Similarly, if the damping is increased three times, then the settling time reduces from 30 seconds to 20 seconds.

Figure:28. Response of the stories 1st to 88th with 3 times absorber damping.

When the damping is increased ten times then there are no vibrations after the first shock and the building comes to rest immediately.

Effects of Changing the Stiffness:

The structural stiffness's have their limits. In our case the stiffness of our building (masses ‘m1’ and ‘m3’) is limited by the material used for its construction, for example the steel, concrete, iron and other constructing materials will have their own values of stiffness’s defined. This in turn limits the overall stiffness for our building. So the only option we are left with is to change the stiffness of our damper, TMD.

The effects of increasing the stiffness is same as that of increasing the damping. When the stiffness is increased ten times, then there are no vibrations and the building comes to rest immediately after the first shock.

Figure:29. Response of the stories 1st to 88th with ten times absorber stiffness.

Amplitude Ratio:

For three degree of freedom system (the system with two masses) considering the lumped mass of the building as ‘m1’ and mass of the damper as ‘m2’, the change in amplitude ratio is discussed with the changing frequency ratio. The amplitude ratio is given by “X/X.st”. Its behavior w-r-t the changing value of the frequency ratio i.e. ‘g=W/Wn’ is plotted in the following graph. The graph is plotted at different values of zeta, the damping ratio.
The amplification factor decreases with the increase in the value of zeta. The first curve is for 'zeta=0.5' and the last curve at the bottom is for 'zeta=2.5'.

Mass Ratio:

For three degree of freedom system (two mass system), considering the lumped mass of the building as 'm1' and mass of the damper as 'm2', the amplitude ratio, 'X/X.st' changes when the mass ratio. Mass ratio is the ratio between the mass of the damper and the mass of the building given by 'mu=m2/m1'.

The rest of the system matrices also follow a particular pattern and thus can be easily written for five or higher number of masses.

Similarly, just with a few changes all the responses which are calculated for a three mass system can be calculated for a five mass system. For example;

This graph shows that the amplification factor decreases with the increase in value of 'mu' at first till it reaches a minimum value. For frequency ratio greater than '0.9' the amplification factor value increases with the increase in mass ratio.

**FIVE MASSES**

Using the state space for the solution of this problem has made it a lot easier to solve even for the higher degrees of freedom. The system defined in state space follows a particular pattern, thus making it easier to write the 'matrix A', the state matrix.

Give below are the state matrices for three masses and five masses respectively;

**3 Masses:**

\[
A = [0 0 0 1 0 0; 0 0 0 0 1 0; 0 0 0 0 0 1; -k1/m1 k1/m1 0 -c1/m1 c1/m1 0; k1/m2 (-k2-k1)/m2 k2/m2 c1/m2 (-c1-c2)/m2 +c2/m2 0 k2/m3 (-k3-k2)/m3 0 c2/m3 (-c3-c2)/m3];
\]

**5 Masses:**

\[
A = [0 0 0 0 1 0 0 0 0; 0 0 0 0 0 1 0 0 0; 0 0 0 0 0 0 1 0 0; 0 0 0 0 0 0 0 1 0; 0 0 0 0 0 0 0 0 1; -k1/m1 k1/m1 0 0 0 -c1/m1 c1/m1 0 0; k1/m2 (-k2-k1)/m2 k2/m2 0 0 c1/m2 (-c1-c2)/m2 +c2/m2 0 0; 0 k2/m3 (-k3-k2)/m3 k3/m3 0 0 c2/m3 (-c3-c2)/m3 c3/m3 0 0; 0 0 k3/m4 (-k4-k3)/m4 k4/m4 0 0 c3/m4 (-c3-c4)/m4 c4/m4; 0 0 k4/m5 (-k5-k4)/m5 k5/m5 0 0 0 c4/m5 (-c4-c5)/m5];
\]
Earthquake Response:

To check the response of a five mass system, the same approximate seismic input as used for three mass system as shown in figure:23, is applied for three seconds. The results are plotted against time and are given below;

Figure:32. Response of stories from 95th to 101st.

Figure:33. Response of stories from 88th to 95th.

Figure:34. Response of Absorber.

Figure:35. Response of stories from 44th to 88th.

Figure:36. Response of stories from 1st to 44th.

The same discussion as done for the three degree of freedom could be made here. The concept of the first shock is also valid here. The behavior pattern is also the same. The difference now is that the mass of the building has been divided into four parts instead of two and the results are more accurate for each section.
CONCLUSIONS

1. As long as the input force is there the damper does not work effectively. But as soon as the force is removed it brings the building to rest in the minimum time possible.

2. If the structural limit of the building is reached before the time the force stops acting, it can cause damage to the structure this can result in the destruction of the building in extreme cases.

3. The tall buildings are very open to the atmospheric disturbances due to high altitude e.g. strong wind gales etc. TMD’s work effectively to damp out these induced vibrations.

4. Increasing the damping of the TMD decreases the settling time and magnitude of oscillations.

5. Increasing the stiffness of the TMD decreases the settling time and magnitude of oscillations.

6. One thing is to be noted here that neither by increasing the damping or the stiffness the time for the first shock is not decreased even though the response becomes zero immediately after the first shock. On the other hand, we can see a decrease in the magnitude by increasing either the stiffness or the damping.

7. Using the state space makes it very easy to increase the number of masses equal to the total floors and to study their responses. The reason is all the system matrices follow a particular pattern as described for three and five masses. This pattern can be used to easily write the code for 102 masses, 101 stories and one damper. But the limitations come in modeling the system and deriving the equation of motions. If this step is done correctly, then the degree of freedoms can be increased up to the desired number.

REFERENCES


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